FACULTY OF MECHANICAL ENGINEERING	institute of solid mechanics, mechatronics and biomechanics	
Experimental mechanics (REM)		
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Elaboration date:	17.1.2019	
Assigment name:	**Combined stress**	
Supervisor	Ing. Petr Krejčí, Ph.D.	Evaluation:

Assignment

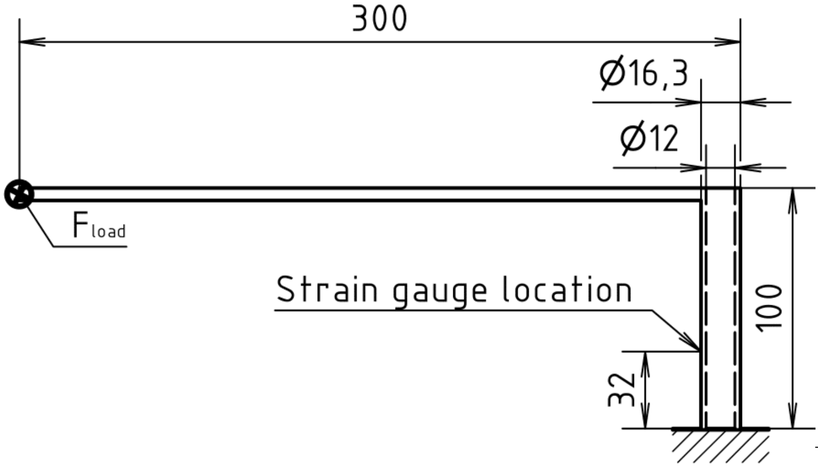
Identify magnitude of bending and torque at the point where strain gauges are glued

1. Analytically (resulting internal forces) and equations
2. Experimentally (SG rosette 0°, 45°, 90°)
3. Numerically by FEM

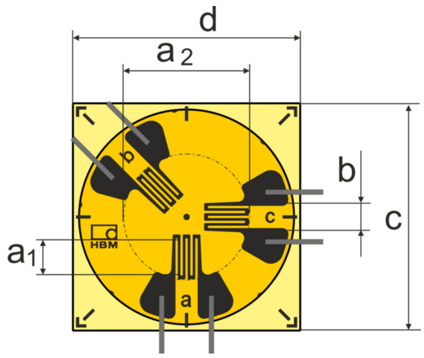
Experiment

Overview

The experiment was performed on apparatus shown below (see fig. 1a). There was single strain gauge rosette for angles 0°, 90° and 45° (see fig. 1b) placed on apperture wired in 1/4 Wheatstone bridges. Used strain gauge amplifier was HBM QuantumX MX1615B.



(a) Apparatus dimensions



(b) SG rosette design[a]

[a]source HBM catalogue

Figure 1: Experiment scheme

Parameters

All needed dimensions of rod are evident from figure 1a.

$$m_{load} = 2010\ g = 2.01\ kg$$
$$F_{load} = m_{load} * g = 2.01 * 9.81 = 19.72\ N$$

Used measuring equipment:

- 4× strain gauge, 120 Ω, Gage Factor 2, 1/4 bridge
- HBM QuantumX MX1615B - Strain gauge amplifier
- HBM U9A – Reference force sensor

Measurement

Data was measured by three strain gauges (see fig. 1b.

Calculation

Analytical solution

Rod params:

$$L = 300\ mm$$
$$h = 100\ mm$$

Cross section params:

$$x_I =< 0; L >$$
$$x_{II} =< 0, h >$$

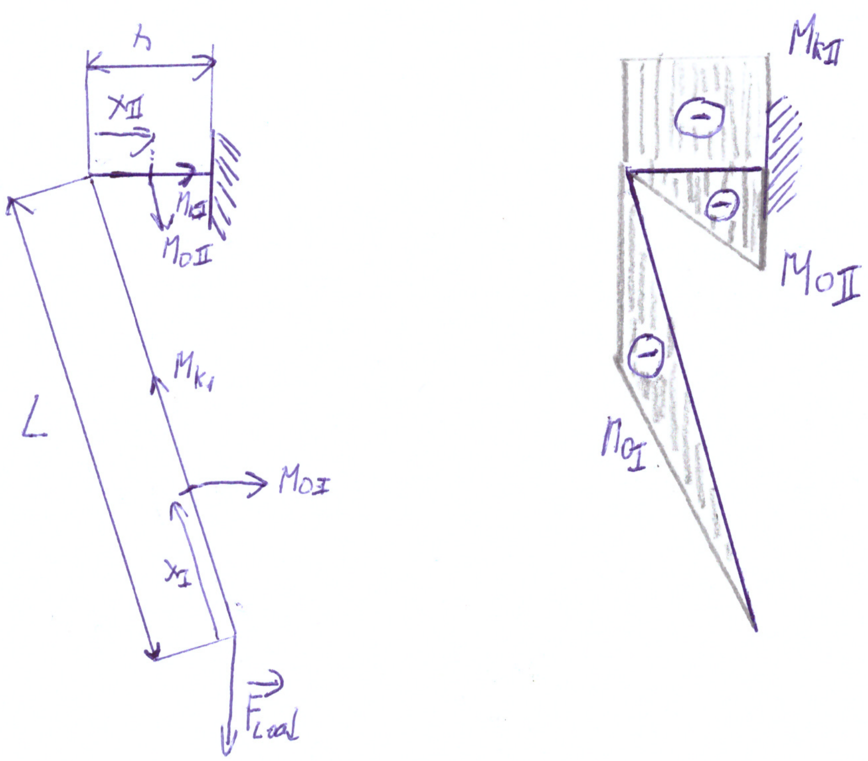


Figure 2: Graph of resulting internal force(s)

Cross section I:

$$M_{bI} = F_{load} * x_I$$
$$M_{tI} = 0$$

Cross section II:

$$M_{bII} = F_{load} * x_{II}$$
$$M_{tII} = F_{load} * L$$

Strain gauge rosette was placed in distance 32 mm from fixed support, which means 68 mm in terms of coordinate x_{II}.
Bending moment M_{bII} and torque M_{tII}:

$$M_{bII} = F_{load} * x_{II} = 19.72 * 0.068 = 1.34\ N * m$$
$$M_{tII} = F_{load} * L = 19.72 * 0.3 = 5.92\ N * m$$

Experimental solution

From measuring, we acqured three deformations.

$$\epsilon_a = -10.5\ {}^{\mu m}/_{m}$$
$$\epsilon_b = -58\ {}^{\mu m}/_{m}$$
$$\epsilon_c = 23.5\ {}^{\mu m}/_{m}$$

- Mohr circle center and radius:

$$C = \frac{\epsilon_a + \epsilon_c}{2} = \frac{-10.5 + 23.5}{2} = 6.5\ {}^{\mu m}/_{m}$$
$$R = \sqrt{(\epsilon_a - C)^2 + (\epsilon_b - C)^2} = \sqrt{(-10.5 - 6.5)^2 + (-58 - 6.5)^2} = 66.7\ {}^{\mu m}/_{m}$$

- Main normal stresses[1]:

$$\sigma_1 = \frac{E}{(1-\mu)} * C + \frac{E}{(1+\mu)} * R = \frac{2.1 * 10^5}{(1-0.3)} * 6.5 + \frac{2.1 * 10^5}{(1+0.3)} * 66.7 = 12.72\ Mpa$$
$$\sigma_2 = \frac{E}{(1-\mu)} * C - \frac{E}{(1+\mu)} * R = \frac{2.1 * 10^5}{(1-0.3)} * 6.5 - \frac{2.1 * 10^5}{(1+0.3)} * 66.7 = -8.82\ MPa$$

- Bending stress, torque stress:

$$\sigma_b = \sigma_1 + \sigma_2 = 12.72 - 8.82 = 3.9\ MPa$$
$$\tau_t = \sqrt{(\sigma_1 - \frac{\sigma_b}{2})^2 - (\frac{\sigma_b}{2})^2} = \sqrt{(12.72 - \frac{3.9}{2})^2 - (\frac{3.9}{2})^2} = 10.59\ Mpa$$

- Bending moment:

$$M_b = \sigma_b * W_o = \sigma_b * \frac{\pi * (D^4 - d^4)}{32 * D} = 3.9 * 10^6 * \frac{\pi * (0.0163^4 - 0.012^4)}{32 * 0.0163} = 1.17\ N * m$$

- Torque:

$$M_t = \tau_t * W_k = \tau_t * \frac{\pi * (D^4 - d^4)}{16 * D} = 10.59 * 10^6 * \frac{\pi * (0.0163^4 - 0.012^4)}{16 * 0.0163} = 6.36\ N * m$$

FEM Solution

Ansys[2] was used to create model (see fig. 3). Instead of mass, force was used (mark A), for apperture basement fixed support was used (mark B). TO determine moments at place of strain gauge rosette, `Moment Reaction` with adequate meshing type had to be used (see fig. 4). Note that moment gathered from ANSYS had orientation in space and had to be disassembled to it's components.
Results according to ANSYS:

$$M_b = 1.31\ N * m$$
$$M_k = 5.92\ N * m$$

[1]For Yung's module and Poisson's ratio structural steel parameters were used
[2]Due to buggy unstable environment under ArchLinux i was forced to use Workbench

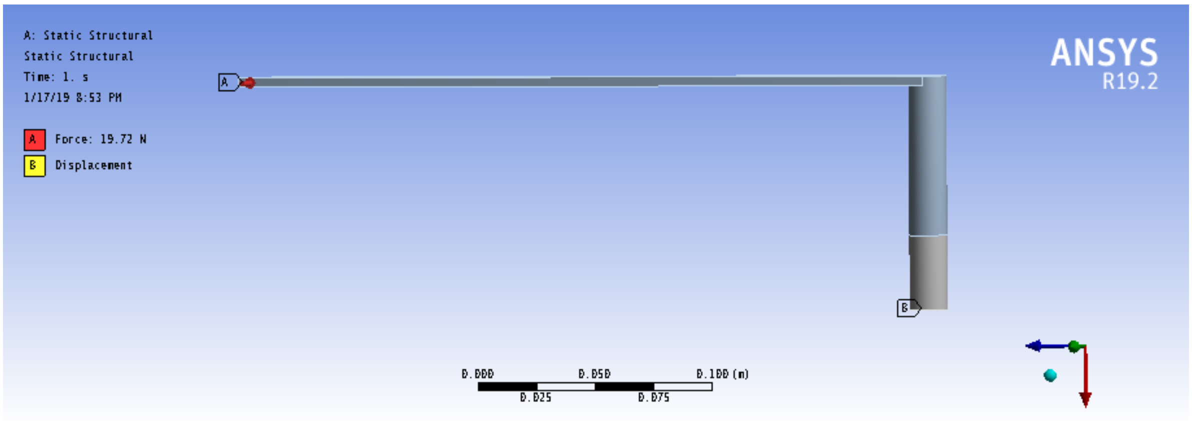


Figure 3: Model of apperture

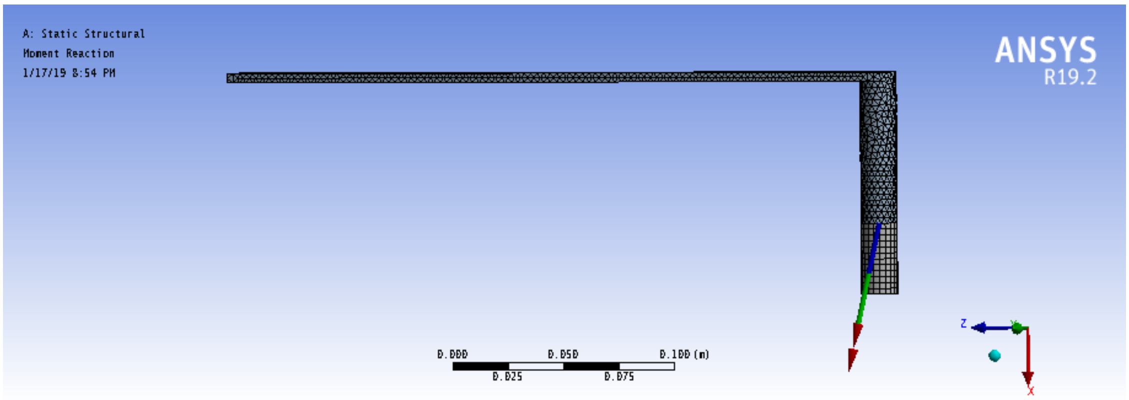


Figure 4: Total resulting moment(s) in place of SG rosett. Mesh detail.

Conclusion

Type \Quantity	M_b $[N * m]$	M_k $[N * m]$
Analytical	1.34	5.92
Experimental	1.17	6.36
Numerical	1.31	5.92

Table 1: Results comparison

We conducted internal resulting moment by three different methods (analytically, experimentally and numerically). Suprisingly, analytical and numerical solution gave very similar results. Sadly this is not truth for experimental measuring.

There could be errors during measurement same as rounding error while calculating intermediate results.