 FACULTY institute OF MECHANICAL of solid mechanics, ENGINEERING mechatronics and biomechanics		
Experimental mechanics (REM)		
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Assignment name:	Broken forcemeter	
Supervisor	Ing. Petr Krejčí, Ph.D.	Evaluation:

Assignment

Identify magnitude of the "force-meter" load by measuring it's deformation. Compare calculated magnitude of the force with measured one. In case of difference bigger than 10% make justification of this difference using FEM¹.

Experiment

Overview

The experiment was performed on the apparatus as shown below (fig. 1a). Dimensions of so called dural force meter are evident fom figure 1b, same as position of strain gauges. There are installed two pairs of strain gauges, one in front of measured part, second one on rear side. All strain gauges were connected as 1/4 bridge, used strain gauge amplifier was QuantumX MX1615B. Sensor used for measuring reference force was HBM U9A.

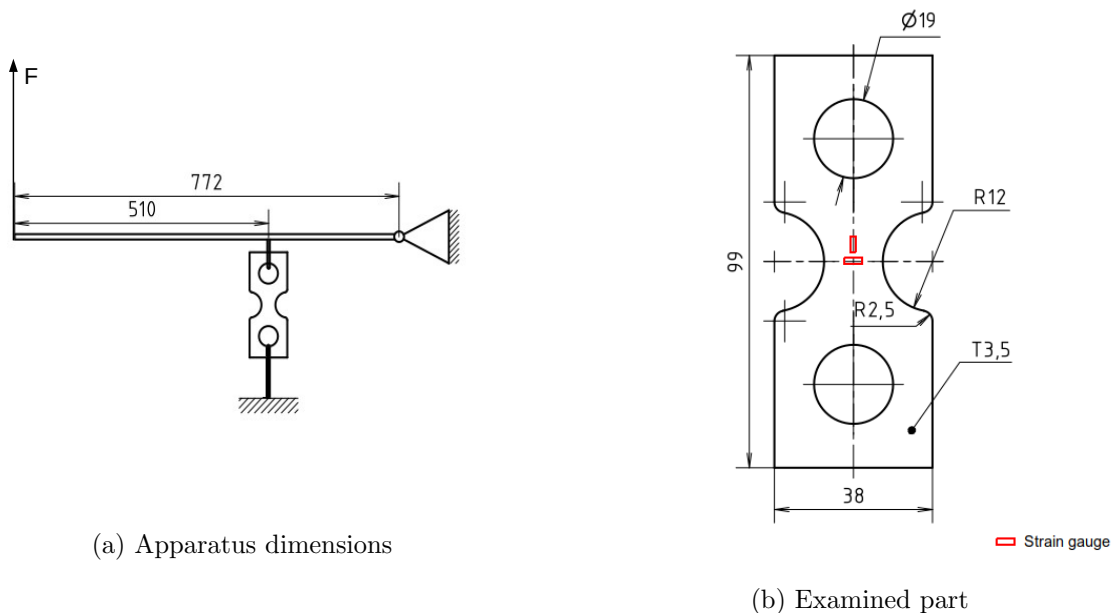


Figure 1: Experiment scheme

¹Finite Element Method

Parameters

All needed dimensions of mechanical parts are evident from figure 1.

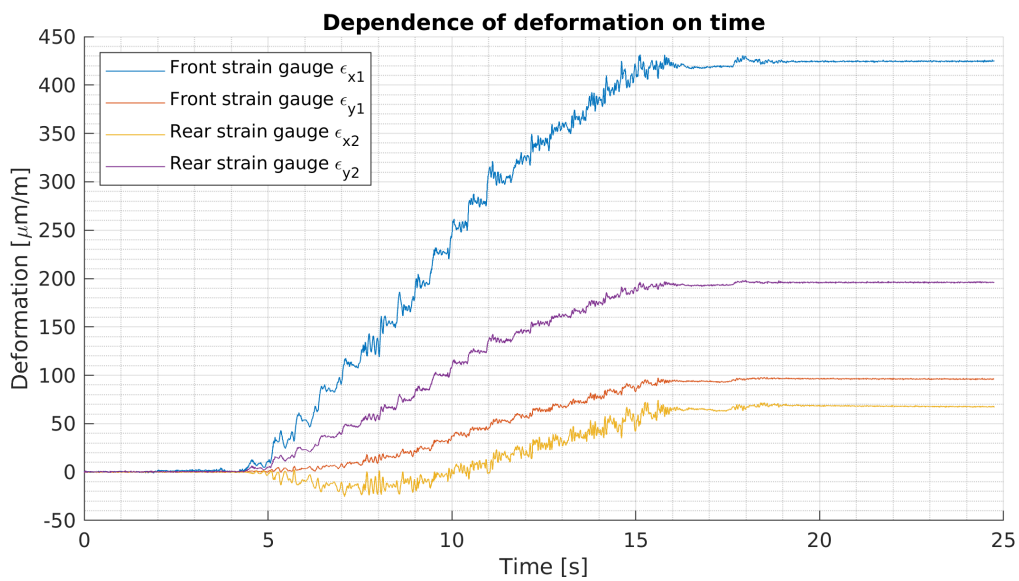
Dural material properties:

$$E = 70 \quad [GPa]$$

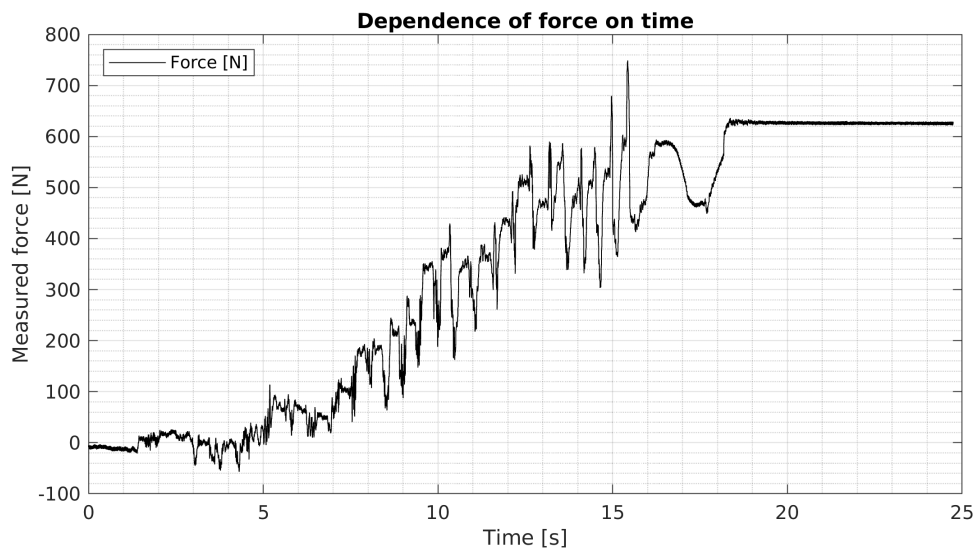
$$\mu = 0.37 \quad [-]$$

Measurement

Strain gauges were placed as seen in figure 1b, wired in 1/4 bridges. Reference force was measured with HBM U9A (see "F" in fig. 1a). Force acting on agent ("force meter") had to be calculated from lever. Values used for calculations are chosen for $t > 20s$ (steady state).



(a) Deformation over time



(b) Force over time

Figure 2: Measured data

Strain data obtained:

$$\begin{aligned}\epsilon_{y1} &= 424.5 \quad [\mu m/m] \\ \epsilon_{x1} &= 96.2 \quad [\mu m/m] \\ \epsilon_{y2} &= 67.9 \quad [\mu m/m] \\ \epsilon_{x2} &= 196.1 \quad [\mu m/m]\end{aligned}$$

Force magnitude:

$$\begin{aligned}F_{ref} &= 625.9 \quad [N] \\ F_{load} &= F_{ref} * L / (L - L_1) \\ F_{load} &= F_{ref} * 772 / (772 - 510) \\ F_{load} &= 1844.3 \quad [N]\end{aligned}$$

Calculation

Forces were calculated from strain using Hooks law. There were calculated variants for 1/4 bridge, half bridge and full Wheatstone bridge. Comparison follows at the end of report.

$$\sigma = \epsilon * E \quad (1)$$

$$F = \sigma * S \quad (2)$$

1/4 bridge

Front side:

$$\begin{aligned}\sigma_{y1} &= \epsilon_{y1} * E = 424.5 * 10^{-6} * 70 * 10^9 = 29.7 \text{ MPa} \\ F_{y1} &= \sigma_{y1} * S = 29.7 * 10^6 * (17.5 * 3.5) * 10^{-6} = \underline{1820.1N}\end{aligned}$$

Rear side:

$$\begin{aligned}\sigma_{y2} &= \epsilon_{y2} * E = 67.9 * 10^{-6} * 70 * 10^9 = 4.8 \text{ MPa} \\ F_{y2} &= \sigma_{y2} * S = 29.7 * 10^6 * (17.5 * 3.5) * 10^{-6} = \underline{291.4N}\end{aligned}$$

1/2 bridge

Front side:

$$\begin{aligned}\epsilon_f &= \frac{\epsilon_{y1} - \epsilon_{x1}}{1.3} = \frac{424.5 - 96.2}{1.3} = 252.5 \\ \sigma_f &= \epsilon_f * E = 252.5 * 10^{-6} * 70 * 10^9 = 17.7 \text{ MPa} \\ F_f &= \sigma_f * S = 29.7 * 10^6 * (17.5 * 3.5) * 10^{-6} = \underline{1082.7N}\end{aligned}$$

Rear side:

$$\begin{aligned}\epsilon_r &= \frac{\epsilon_{y2} - \epsilon_{x2}}{1.3} = \frac{67.9 - 196.1}{1.3} = -98.6 \\ \sigma_r &= \epsilon_r * E = 2952.5 * 10^{-6} * 70 * 10^9 = -68.9 \text{ MPa} \\ F_r &= \sigma_r * S = 29.7 * 10^6 * (17.5 * 3.5) * 10^{-6} = \underline{-422.4N}\end{aligned}$$

Full Wheatstone bridge:

$$\begin{aligned}\epsilon_w &= \frac{\epsilon_{y1} - \epsilon_{x1} + \epsilon_{y2} - \epsilon_{x2}}{2.3} = \frac{424.5 - 96.2 + 67.9 - 196.1}{2.3} = 76.9 \\ \sigma_w &= \epsilon_w * E = 252.5 * 10^{-6} * 70 * 10^9 = 53.8 \text{ MPa} \\ F_w &= \sigma_w * S = 29.7 * 10^6 * (17.5 * 3.5) * 10^{-6} = \underline{330.1N}\end{aligned}$$

FEM Solution

Model of measured element was made in ANSYS FEM software. To increase model precision, little surfaces were created in mount holes in places, where loads were situated.

One area was loaded with force equivalent to F_{load} , second one was equipped with fixed support. Material properties were set to approximate dural. For more details, see figurures 3, 4, 5, 6 and 7.

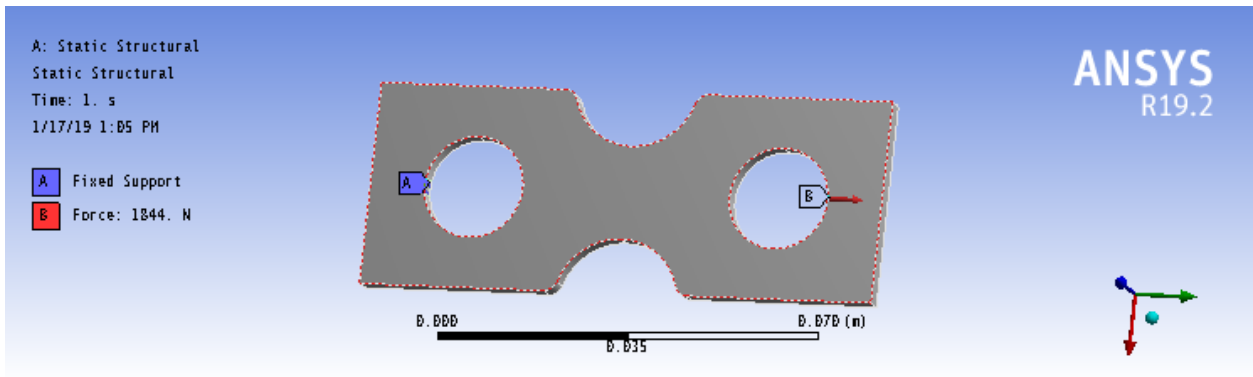


Figure 3: Model of force meter with loads

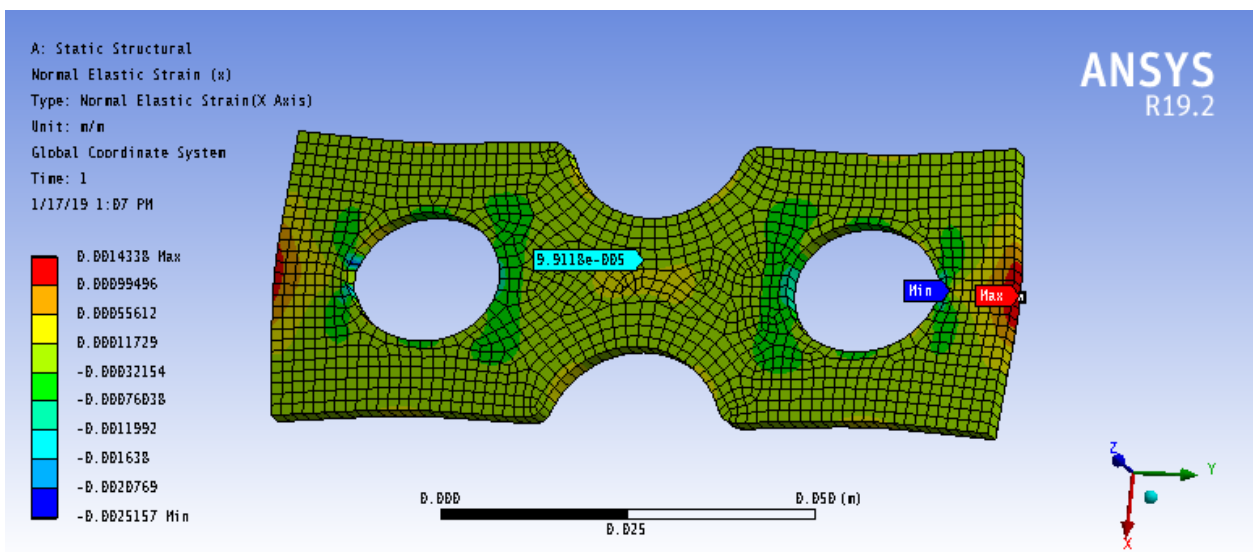


Figure 4: Normal strain in axis X

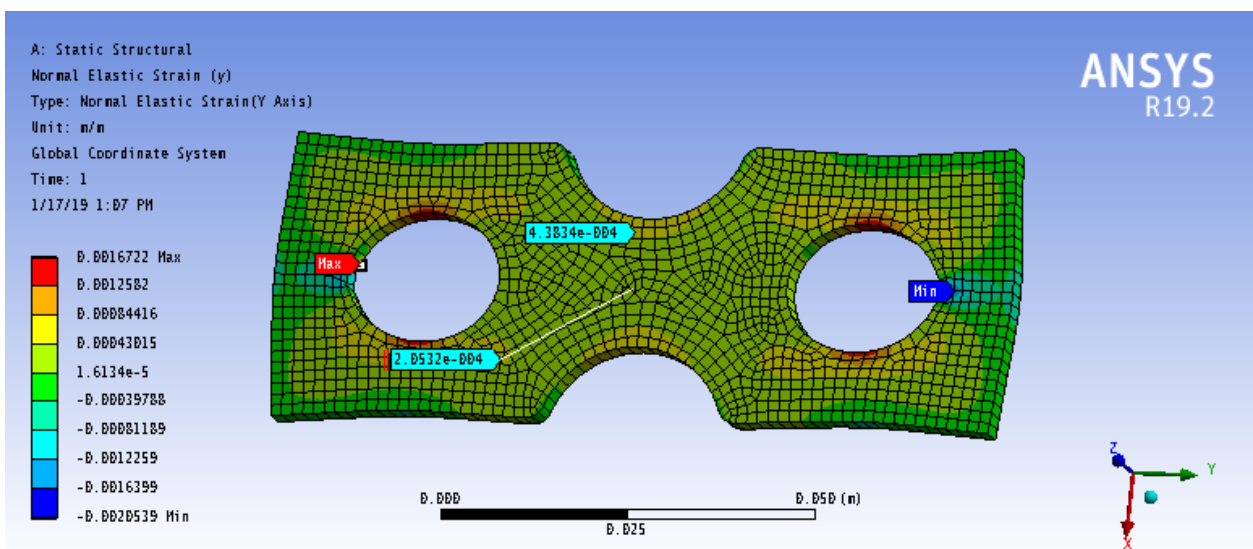


Figure 5: Normal strain in axis Y

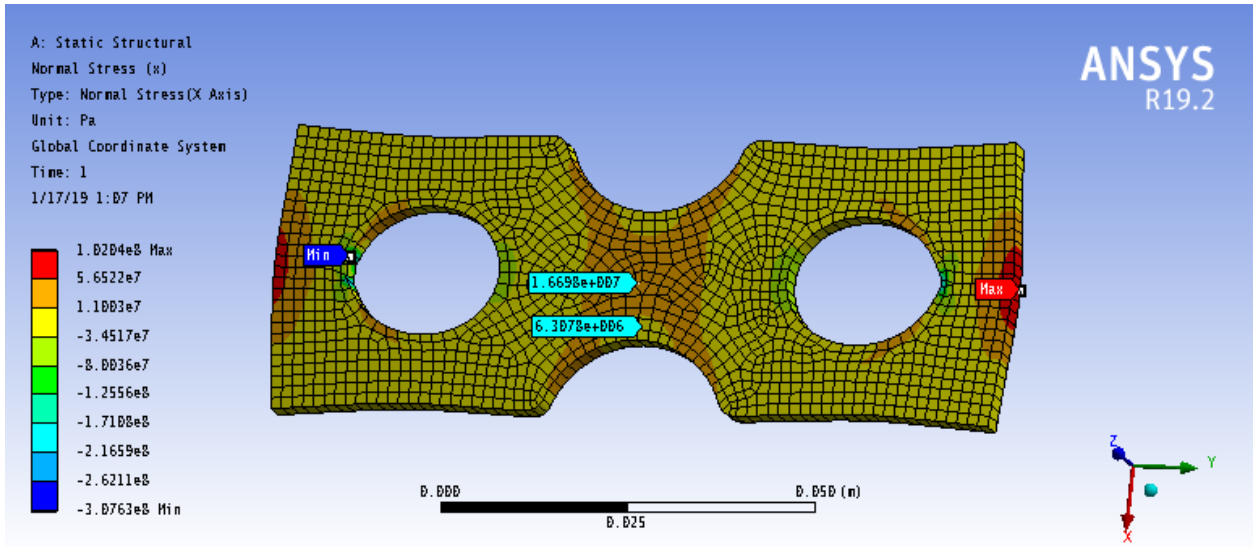


Figure 6: Normal stress in axis X

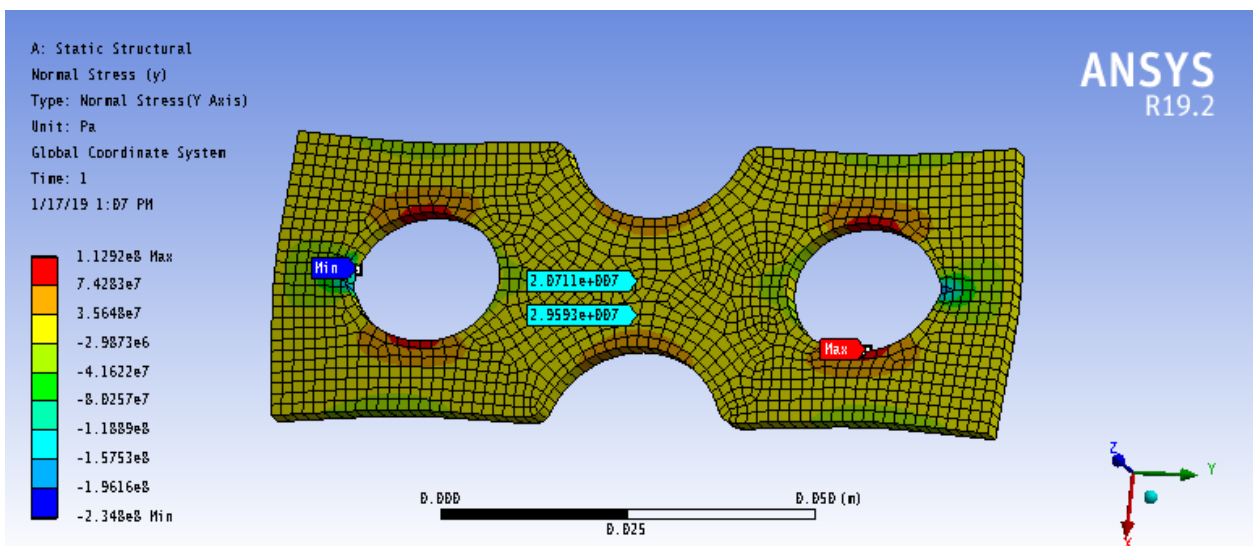


Figure 7: Normal stress in axis Y

Below are summarised data gathered from ANSYS. Results were pinned from coordinates where strain gauges were placed on real part. Nice thing is, that highlighted values are roughly approximate to measurement data.

Normal strain:

$$\epsilon_y = 2.053 * 10^{-4} [m/m] = 205.3 [\mu m/m]$$

$$\epsilon_x = 9.91 * 10^{-5} [m/m] = 99.1 [\mu m/m]$$

Normal stress:

$$\sigma_y = 20.7 MPa$$

$$\sigma_x = 16.6 MPa$$

Conclusion

Variable \ Data type	Measured	Analytical	FEM
F	1844	-422.4 ... 1820	-
ϵ_y	424.5	-	205.3
ϵ_x	96.2	-	99.1
σ_y	-	29.7	20.7
σ_x	-	6.73	16.6

Table 1: Data resume

After better look at table 1, we can recognize that measured, analytically calculated and FEM simulated data differ in better case in tens of percent, in worse case is one column multiple of another one. This can have multiple causes. In my opinion, one of most significant factors is theory applied on analytical solution. Theory expect constant cross-section (it's area) around location of strain gauges, but in real, there are two D-shaped cutouts at that place. This is noticeable from FEM screenshots. Another cause can be slightly different shape and material of measured force meter or imperfect experiment environment. Last, but not least parameter is possible bad use of FEM, which is numerical method - and that means it's error prone.

Biggest difference can be seen in normal strain and stress (see. table 1). In case of strain we can determine by the look at data. Front and rear side differs a lot. That difference should not happen - so it is safe to say that "force meter" is not perfect rod and may also be bent around axis X.

In case of normal stress we can behold two problems. At first, we use wrong equations to calculate stress. As seen on picture, normal stress is distributed in two axis and not one which means we should not use Hook's law for its calculation. In the other hand we are using measured strain data that, as mentioned above, may not be reliable.