$$y_h = c_1 \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} e^x + c_2 \begin{pmatrix} -2\sin(2x) \\ \cos 2x \\ 3\cos 2x \end{pmatrix} e^x + c_3 \begin{pmatrix} 2\cos 2x \\ \sin 2x \\ 3\sin 2x \end{pmatrix} e^x$$

$$f = \begin{pmatrix} \frac{3 + \sin 2x - 6e^x \sin 2x - x^3}{2} \\ \\ e^x + 2x\sin 2x \end{pmatrix} = \underset{\lambda^*=0}{\overset{0:}{\begin{pmatrix} 3x^2 \\ 2 \\ 0 \end{pmatrix}}} + \underset{\lambda^*=2i}{\overset{2i}{\begin{pmatrix} \sin(2x) \\ 0 \\ 2x\sin 2x \end{pmatrix}}} + \underset{\lambda^*=1+2i}{\begin{pmatrix} -6e^x\sin 2x \\ 0 \\ 0 \end{pmatrix}} + \begin{pmatrix} 0 \\ 0 \\ e^{-x} \end{pmatrix}$$

$$y_{p1} = \begin{pmatrix} A_1x^3 + B_1x^2 + C_1 + D_1 \\ A_2x^3 + B_2x^2 + C_2 + D_2 \\ A_3x^3 + B_3x^2 + C_3 + D_3 \end{pmatrix} \quad y_{p2} = \begin{pmatrix} (Ax+B)\sin(2x) + (Cx+D)\cos 2x \\ (Ex+F)\sin 2x + (Gx+H)\cos 2x \\ (Ix+J)\sin 2x + (Kx+L)\cos 2x \end{pmatrix} \quad y_{p3} = \begin{pmatrix} (Ax+B)\sin(2x) + (Cx+D)\cos 2x \\ \\ \\ \end{pmatrix} e^x \quad y_{p4} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}$$

ŘEŠENÍ ODR POMOCÍ MOCNINNÝCH ŘAD

Př: $3y' + 2yx = 1 \; ; \; y(0) = 2$

předpoklad tvaru řešení: $y(x) = \sum_{k=0}^{\infty} a_k \cdot (x - x_0)^k \; ; \; a_k = \frac{y^{(k)}(x_0)}{k!}$

střed ve kterém je dána poč. podmínka, $x_0 = 0$

$$y' = \frac{1}{3}(1 - 2yx)$$

$$y'' = -\frac{2}{3}(y'x + y) = -\frac{2}{3}(2) = -\frac{4}{3} = y''(0)$$

$$y''' = -\frac{2}{3}(y''x + 2y') = \frac{2}{3}\left(\frac{2}{3}\right) = \frac{-4}{9} = y'''(0)$$

k	a_k	$y^{(k)}$
0	2	2
1	1/3	1/3
2	-2/3	4/3
3	-4/54	-4/9

$a_1 = \frac{y'(0)}{1!} = \frac{1}{3}(1 - 2\cdot 2\cdot 0) = \frac{1}{3}$

$a_2 = \frac{y''(0)}{2!} = \frac{4/3}{2} = -\frac{2}{3}$

$$y(x) \approx 2 + \frac{1}{3}x - \frac{2}{3}x^2 - \frac{4}{54}x^3$$

Př: $y' = 3y \; ; \; y(0) = 1$

$y(x) = \sum a_k x^k$

$y' = 3y$

$y'' = 3y'$

$y''' = 3y''$

$y^{(k+1)} = 3y^{(k)}$

$a_k = \frac{y^{(k)}(0)}{k!}$

$a_{k+1} = \frac{3}{(k+1)} a_k \quad \forall k = 1,2,3\ldots$

$a_0 = 1$

$a_1 = \frac{3}{1} \cdot 1 = 3$

$a_2 = \frac{3}{2} \cdot 3 = \frac{9}{2}$

$a_3 = \frac{3}{3} \cdot \frac{9}{2} = \frac{9}{2}$

$$\left.\right\} a_k = \frac{3^k}{k!} \Rightarrow \sum_{k=0}^{\infty} \frac{3^k}{k!} x^k = e^{3x}$$