

# nehomogenní SLODR

Pr.  $y' = \begin{pmatrix} -1 & 2 \\ -2 & -6 \end{pmatrix} y + \begin{pmatrix} 7 \\ -14 \end{pmatrix} e^{2x} + \begin{pmatrix} -20 \\ 0 \end{pmatrix}$ ;  $y(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

① přidružení

$$y' = Ay$$

vl. č.:  $\begin{vmatrix} -1-\lambda & 2 \\ -2 & -6-\lambda \end{vmatrix} = \lambda + 7\lambda + 10 = 0$   $\lambda_1 = -5$   
 $(\lambda + 5)(\lambda + 2) = 0$   $\lambda_2 = -2$

$\lambda_1 = -5$

$$\begin{vmatrix} -1-(-5) & 2 \\ -2 & -6-(-5) \end{vmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0$$

h.s.:  $\begin{pmatrix} 4 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0$   $4h_1 + 2h_2 = 0$   $h_1 = -\frac{h_2}{2}$   $v: h_1 = 1$   $h_2 = -2$   $h = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$u_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-5x}$

$\lambda_2 = -2$

$$\begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} h_1^* \\ h_2^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} h_1^* \\ h_2^* \end{pmatrix} = 0$$
  $h_1 + 2h_2 = 0$   $h_1 = -2h_2$   $v: h_2 = 1 \Rightarrow h_1 = -2$   $h^* = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$u_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-2x}$

$$y_h = c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-5x} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-2x}$$

$f_1 = \begin{pmatrix} 7 \\ -14 \end{pmatrix} e^{2x}$

$a = 2$

$b = 0$

$y_{p1} = \begin{pmatrix} A \\ B \end{pmatrix} e^{2x}$

$y_{p1}' = \begin{pmatrix} 2A \\ 2B \end{pmatrix} e^{2x}$

dosadíme do rovnice:  $y_p$

$A = \begin{pmatrix} -1 & 2 \\ -2 & -6 \end{pmatrix}$

$y_{p1}' = Ay + f_1$

$2Ae^{2x} = -Ae^{2x} + 2Be^{2x} + 7e^{2x}$

$2Be^{2x} = -2Ae^{2x} + 6Be^{2x} - 14e^{2x}$

~~$3A - 2B = 7$~~   
 ~~$2A - 4B = -14$~~   $(-3)$

$3A - 2B = 7$   $(2)$

①  $A + 4B = -7$

$7A = 7 \Rightarrow A = 1$

$4B = -8 \Rightarrow B = -2$

$y_{p1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{2x}$

$f_2 = \begin{pmatrix} -20 \\ 0 \end{pmatrix}$

$y_{p2} = \begin{pmatrix} A \\ B \end{pmatrix} e^0$

$y_{p2}' = (0) \cdot 1$

dosadíme do  $y_{p2}' = Ay + f_2$

$0 = -A + 2B - 20$   $(-2)$

$0 = -2A - 6B$

$0 = -10B + 40 \Rightarrow B = 4$

$A = -12$

$y_{p2} = \begin{pmatrix} -12 \\ 4 \end{pmatrix}$

$y = y_h + y_{p1} + y_{p2}$

$y = c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-5x} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-2x} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{2x} + \begin{pmatrix} -12 \\ 4 \end{pmatrix}$