EXAKTNÍ ROVNICE

PF $\underbrace{\frac{2x}{y^3}}\,dx + \underbrace{\frac{y^2-3x^2}{y^4}}\,dy = 0$

$\underbrace{P(x,y)\,dx + Q(x,y)\cdot dy}_{dF(x,y)} = 0$

$$\frac{\partial F}{\partial x} = P \quad , \quad \frac{\partial F}{\partial y} = Q$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x}$$

$$\frac{2x}{y^3} + \frac{y^2-3x^2}{y^4} y' = 0$$

$$y' = \frac{-2x}{y^3} \cdot \frac{y^4}{(y^2-3x^2)}$$

ověřit, že parc. podle y = parc der podle x

OVĚŘ, ŽE $\cancel{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$

$$\frac{\partial P}{\partial y} = \frac{\partial (2xy^{-3})}{\partial y} = -6y^{-4}x$$

$$\frac{\partial Q}{\partial x} = \frac{\partial (y^2-3x^2)}{\partial y^4} = -6y^{-4}x$$

= rovnají se

JEDNÁ SE O EXAKTNÍ RCI

$\forall (x,y) \in \mathbb{R}^2 \setminus \{y=0\}$

Hledáme řešení ve tvaru $F(x,y) = C$

$$F(x,y) = \int P(x,y)\,dx = \int \frac{2x}{y^3} \div \varphi(y)$$

$$\frac{\partial F}{\partial y} = Q : \quad \underbrace{\frac{-3x^2}{y^4} + \varphi'(y)}_{\frac{\partial F}{\partial y}} = \underbrace{\frac{y^2}{y^4} - \frac{3x^2}{y^4}}_{Q}$$

$$\frac{d\varphi}{dy} = y^{-2}$$

$$\int d\varphi = \int y^{-2}\,dy$$

$$\varphi = \frac{-1}{y} + D; \ D \in \mathbb{R}$$

$$F(x,y) = \frac{x^2}{y^3} - \frac{1}{y} + D; \ D \in \mathbb{R}$$

obecné řešení: $\frac{x^2}{y^3} - \frac{1}{y} = C$; $C \in \mathbb{R}$