LAPLACEOVA TRANSFORMACE

$$\boxed{\mathcal{L}\{f(x)\}(s) = \int_0^\infty f(x)\, e^{-sx}\, dx}$$

$f(x)$	$F(s)$
1	$1/s$
x	$1/s^2$
x^n, $n\in\mathbb{N}$	$n!/s^{n+1}$
e^{ax}, $a\in\mathbb{R}$	$1/(s-a)$
$x^n e^{ax}$	$n!/(s-a)^{n+1}$
$\sin(ax)$	$a/(s^2+a^2)$
$\cos(ax)$	$s/(s^2+a^2)$
$f'(x)$	$sF(s)-f(0^+)$
$e^{ax}\cdot\sin(bx)$	$b/[(s-a)^2+b^2]$
$e^{ax}\cdot\cos(bx)$	$(s-a)/[(s-a)^2+b^2]$
$f^{(n)}(x)$	$s^nF(s)-s^{n-1}f'(0+)-\ldots-s^{n-2}f(0+)-f(0+)$

$$\text{ODR} \xrightarrow{\mathcal{L}} \text{ALG. R-CE}$$

$$y(x) \xleftarrow{\mathcal{L}^{-1}} \varphi(s)$$

Př. Odvoďte Laplaceův obraz fce $f(x)=e^{x-\tau}$

$$F(s)=\mathcal{L}\{f(x)\}(s)=\int_0^\infty e^{x-\tau}\cdot e^{-sx}\,dx = e^{-\tau}\int_0^\infty e^{(1-s)x}\,dx =$$

$$= e^{-\tau}\cdot\lim_{b\to\infty}\left[\frac{e^{(1-s)x}}{1-s}\right]_0^b = \left|e^{-\tau}\cdot\left(-\frac{1}{(1-s)}\right)\right| = \frac{e^{-\tau}}{s-1}$$

Př. $y'+3y=2$; $y(0)=25$

Laplace:

$$\mathcal{L}\{y'(x)+3y(x)\}(s)=\mathcal{L}\{2\}(s) \quad \Big|\ \mathcal{L}\{y(x)\}(s)=Y(s)$$

$$s\cdot Y(s) - y(0+) + 3Y(s) = \frac{2}{s}$$

$$s\cdot Y(s) - 25 + 3Y(s) = \frac{2}{s}$$

$$(s+3)Y(s) = \frac{2+25s}{s}$$

$$Y(s)=\frac{2+25s}{s(s+3)} = \frac{A}{s}+\frac{B}{(s+3)} \qquad = \frac{2}{3s}+\frac{73}{3(s+3)}$$

$$2+25s = As+3A+sB$$

s^1: $25 = A+B$

s^0: $2=3A \Rightarrow \underline{A=\frac{2}{3}}$

$$B = 25-\frac{2}{3} = \frac{73}{3}$$

$$y(x)=\mathcal{L}^{-1}\{Y(s)\}(x) = \frac{2}{3}+\frac{73}{3}e^{-3x}$$

? a proč ne tohle ?

Př. $y''-2y'+5y=0$

$$s^2Y(s) - sy(0+) - y'(0+) - 2[sY(s)-y(0+)] + 5Y(s) = 0$$

ozn: $y(0+)=c_1$; $y'(0+)=c_2$

$$s^2Y(s) - sc_1 - c_2 - 2sY(s) + 2c_1 + 5Y(s) = 0$$

$$Y(s)\cdot(s^2-2s+5) = sc_1+c_2-2c_1$$

$$Y(s) = \frac{sc_1+c_2-2c_1}{(s^2-2s+5)} \rightarrow s_{1,2}=\frac{2\pm\sqrt{4-4\cdot5}}{2a}$$

X nejde rozložit ⇒ rozklad na čtverec

$$s^2-2s+5 = (s-1)^2+4$$

$a=2$, $b=2$?

$$Y(s) = \frac{c_1(s-2)}{(s-1)^2+4} + c_2\frac{1}{(s-1)^2+4}$$

$$Y(s) = \frac{c_1(s-1)}{(s-1)^2+4} + \frac{(c_2-c_1)}{2}\cdot\frac{1}{(s-1)^2+4}$$

$$y = \underset{D_1}{C_1}\cdot e^x\cos(2x) + \underset{D_2}{\frac{c_2 c_1}{2}}\, e^x\sin(2x)$$