

PF: $y' = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} y + f$

M.C.: $\begin{vmatrix} 1-\lambda & -1 & -1 \\ 1 & 1-\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 - (1-\lambda) + 3 - 3\lambda = (1-\lambda)(\lambda^2 - 2\lambda + 5)$ $\lambda_1 = 1$ $\lambda^2 - 2\lambda + 5 = \lambda_{2,3} = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$

$\lambda_1 = 1$

$\begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} h_1 = 0 \Rightarrow \begin{pmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} h_1 = 0$ $h_1 = 0$ $h_2 = h_3$ $v: h_2 = 1, h_3 = 1$ $h_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ $u_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^x$

$\lambda_2 = 1 + 2i$

$\begin{pmatrix} 2i & -1 & -1 \\ 1 & -2i & 0 \\ 3 & 0 & -2i \end{pmatrix} h_2 = 0 \Rightarrow \begin{pmatrix} 1 & -2i & 0 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{pmatrix} h_2 = 0 \rightarrow \begin{pmatrix} 1 & -2i & 0 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{pmatrix} h_2 = 0$ $h_1 - 2i h_2 = 0$ $3h_2 = h_3$ $v: h_2 = 1$ $h_3 = 3$ $h_1 = 2i$ $h_2 = \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}$ $u_2 = \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} e^{(1+2i)x}$

$u = \begin{pmatrix} 2i \cos 2x - 2 \sin 2x \\ \cos 2x + i \sin 2x \\ 3 \cos 2x + 3i \sin 2x \end{pmatrix} e^x = \underbrace{\begin{pmatrix} -2 \sin 2x \\ \cos 2x \\ 3 \cos 2x \end{pmatrix} e^x}_{u_2} + \underbrace{\begin{pmatrix} 2 \cos 2x \\ \sin 2x \\ 3 \sin 2x \end{pmatrix} i e^x}_{u_3}$

$y_h = c_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^x + c_2 e^x \begin{pmatrix} -2 \sin 2x \\ \cos 2x \\ 3 \cos 2x \end{pmatrix} + c_3 e^x \begin{pmatrix} 2 \cos 2x \\ \sin 2x \\ 3 \sin 2x \end{pmatrix}$

proci se sem mri repi se i?

$f = \begin{pmatrix} 3 + \sin 2x - 6e^x \sin 2x - x^3 \\ 2 \\ e^{-x} + 2x \sin 2x \end{pmatrix} = \begin{pmatrix} 0 \\ 3-x^3 \\ 2 \end{pmatrix} \begin{matrix} \lambda^* = 0 \\ \lambda^* = 2i \\ \lambda^* = 1+2i \end{matrix} + \begin{pmatrix} \sin 2x \\ 0 \\ 2x \sin 2x \end{pmatrix} \begin{matrix} \lambda^* = 2i \\ \lambda^* = 1+2i \end{matrix} + \begin{pmatrix} -6e^x \sin 2x \\ 0 \\ 0 \end{pmatrix} \begin{matrix} \lambda^* = 1+2i \end{matrix} + \begin{pmatrix} 0 \\ 0 \\ e^{-x} \end{pmatrix} \begin{matrix} \lambda^* = 0 \end{matrix}$

$y_{p1} = \begin{pmatrix} A_1 x^3 + B_1 x^2 + C_1 + D_1 \\ A_2 x^3 + B_2 x^2 + C_2 + D_2 \\ A_3 x^3 + B_3 x^2 + C_3 + D_3 \end{pmatrix}$

$y_{p2} = \begin{pmatrix} (Ax+B) \sin(2x) + (Cx+D) \cos(2x) \\ (Ex+F) \sin 2x + (Gx+H) \cos 2x \\ \vdots \end{pmatrix}$

$y_{p3} = y_{p2} \cdot e^x$

$y_{p4} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} e^{-x}$