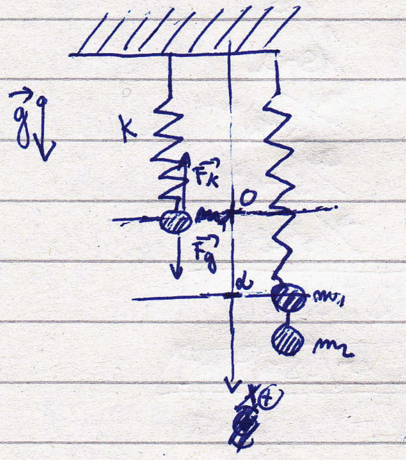
Př. Dvě stejná závaží jsou zavěšena na konci pružiny. Pružina se prodloužila o d [cm] oproti 1 závaží. Určete pohyb 1. závaží, jestliže se 2. ~~...~~ utrhne. Tření zanedbejte a určete dobu kmitu.



$$\vec{F} = m\cdot\vec{a} = \vec{F_k} + \vec{F_g}$$

$$\cancel{F = m}\cdot \quad mg - kx = m\ddot{x}$$

$x = x(t)$ – poloha v čase t

$$m\ddot{x} + kx = mg$$

$$\ddot{x} + \frac{k}{m}x = g$$

podmínky: $x(0) = d$

$\dot{x}(0) = 0$

① přidružená homo

$$\ddot{x}_h + \frac{k}{m}x_h = 0$$

$$\lambda^2 + \frac{k}{m} = 0$$

ozn: $\omega^2 = \frac{k}{m}$

$$\lambda^2 = -\omega^2$$

$$\lambda_{1,2} = \pm\omega i \quad u_1 = \cos(\omega t), \quad u_2 = \sin(\omega t)$$

$$x_h(t) = c_1\cos(\omega t) + c_2\sin(\omega t)$$

② met. neurčitých koef.

$f(t) = g \quad \cdot \quad x_p(t) = A$

$$x_p(t) = 0 = \ddot{x}_p(t)$$

$$0 + \frac{k}{m}\cdot A = g$$

$$A = \frac{mg}{k} = x_p(t)$$

$$x(t) = x_h(t) + x_p(t)$$

$$x(t) = c_1\cos(\omega t) + c_2\sin(\omega t) + \frac{mg}{k}$$

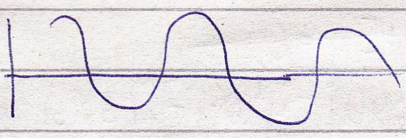
$$\dot{x}(t) = -c_1\omega\sin(\omega t) + c_2\omega\cos(\omega t)$$

$x(0) = d = c_1 + \frac{mg}{k}$

$\dot{x}(0) = 0 = c_2\omega \Rightarrow c_2 = 0$

$c_1 = d - \frac{mg}{k}$

$$x(t) = \left(d - \frac{mg}{k}\right)\cos(\omega t) + \frac{mg}{k}$$



DÚ: $y'' - 8y' + 16y = xe^{2x}$

① $\lambda^2 - 8\lambda + 16 = 0$

$$(\lambda-4)(\lambda-4) = 0$$

$$\lambda_{1,2} = 4 \quad u_1 = e^{4x}, \quad u_2 = e^{4x}$$

$$y_h = c_1e^{4x} + c_2e^{4x}$$

$f(x) = xe^{2x}$ …. $u_{p_1} = e^{2x}\cdot x\cancel{A}$

$a = 2$

$b = 0$

$u_{p_1}' = 2A\cancel{x}e^{2x}\cancel{x} + \cancel{2}Ae^{2x}$

$u_{p_1}'' = 4A\cancel{e^{2x}x} + 2Ae^{2x} + 2Ae^{2x}$

$\lambda = 2 \neq$ jednonásobný kořen

$k = 1$

$\cancel{u_{p_2}''' = 8Ae^{2x}x + 4Ae^{2x} + 8Ae^{2x}}$

$\cancel{4A} - 4A - 8A = x$

$x^1 \Rightarrow 4A = 1 \quad \cancel{A} \frac{1}{4}$

$x^0 \Rightarrow \cancel{8A} = 0$

$$y = c_1e^{4x} + c_2e^{4x} + \frac{1}{4}e^{2x}x$$

$$y'' - 8y' + 16y = xe^{2x}$$

$4Ae^{2x}x + 4Ae^{2x} - 16Ae^{2x}x - 8Ae^{2x} = xe^{2x}$

$e^{2x}x$: $\cancel{4A - 16A = 1}$ $\quad \cancel{16e^{2x}xA}$

$\cancel{-12A = 1}$

$\cancel{A = \frac{1}{12}}$

x^0: $4A - 8A = 0$

$A = 0$