Cramerovo pravidlo : $c_1' = \frac{|W_1|}{|W|}$; $c_2' = \frac{|W_2|}{|W|}$

$$\det(W) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = \underline{\underline{1}}$$

$$\det(W_1) = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\sin x} & \cos x \end{vmatrix} = \underline{\underline{-1}}$$

$$\det(W_2) = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\sin x} \end{vmatrix} = \frac{\cos x}{\sin x}$$

$$c_1'(x) = \frac{|W_1|}{|W|} = \frac{-1}{1} \Rightarrow \frac{dc_1}{dx} = -1$$

$$\int dc_1 = -\int 1\,dx$$

$$\underbrace{c_1(x) = -x + D_1}$$

$$c_2'(x)\ \frac{|W_2|}{W} = \frac{\cos x}{\sin x}$$

$$\int dc_2 = \int \frac{\cos x}{\sin x}\,dx$$

$$\underbrace{c_2 = \ln|\sin x| + D_2}$$

$$\underline{y = (D_1 - x)\cos x + (D_2 + \ln|\sin x|)\sin x}$$

$$\underline{y = \underbrace{D_1 \cos x + D_2 \sin x}_{\text{homogenní}} \underbrace{- x\cos x + \ln|\sin x| \cdot \sin x}_{\text{partikulární}}}$$

$$y = y_h + y_p$$

Př: $f(x) = e^{ax}\left[P_m(x)\cos(bx) + P_m^*(x)\sin(bx)\right]$

$$y_p(x) = e^{ax} x^k \left[Q_M(x)\cos(bx) + Q_M^*(x)\sin(bx)\right]$$

$M = \max\{m, m^*\}$ (stupně polynomu m a m^*) při $b=0$: $m^*=0$

Q_M, Q_M^* – polynomy s neurčitými koeficienty

k – násobnost kořene $\lambda^* = a + bi$ v char. rovnici.

Př: $\underline{y''' - 4y' = e^{2x} + \cos(2x)} = f(x)$ $\quad f_1(x) = e^{2x}$ $\quad f_2 = \cos 2x$

① přidružená homogenní rce

$y_h''' - 4y_h' = 0$

char. pol: $\lambda^3 - 4\lambda = 0$

$\lambda(\lambda - 4) = 0$

$\lambda(\lambda - 2)(\lambda + 2) = 0$

$\lambda_1 = 0 \sim u_1 = e^{0x} = 1$

$\lambda_2 = 2 \sim u_2 = e^{2x}$

$\lambda_3 = -2 \sim u_3 = e^{-2x}$

$\underline{y_h = c_1 + c_2 e^{2x} + c_3 e^{-2x}}$

$f(x) \sim y_p$ $\qquad f(x) = f_1 + f_2$; $y_p = y_{p_1} + y_{p_2}$

$f_1(x) \sim y_{p_1}$

$f_2(x) \sim y_{p_2}$

② $f_1(x) = e^{2x}$

$a = 2$

$b = 0$; $m^* = 0$, $m = 0$

$\underline{M = 0}$

$\lambda^* = 2 + 0i = 2$

– jednoduchý kořen v char pol $\Rightarrow k = 1$

$y_{p_1}(x) = e^{2x} x^1 \left[A \cdot \cos(0x) + B \cdot \sin(0x)\right] \Rightarrow y_{p_1} = e^{2x} x \cdot A$

$y_{p_1}'(x) = Ae^{2x} + 2Ae^{2x}$

$y_{p_1}''(x) = 2Ae^{2x} + 2Ae^{2x} + 4Ae^{2x} \cdot x$

$\underline{y_{p_1}'''(x) = 8Ae^{2x} + 4Ae^{2x} + 8Axe^{2x}}$

$y_{p_1}''' - 4y_{p_1}' = e^{2x}$

$12Ae^{2x} + \cancel{8Axe^{2x}} - 4Ae^{2x} - \cancel{8Axe^{2x}} = e^{2x}$ $\qquad \underbrace{y_{p_1} = \frac{x}{8} e^{2x}}$

$8Ae^{2x} = e^{2x}$

$A = 1/8$