10+)

SLODR$_1$

$$y_1' = 2y_1 + y_2 \qquad y_1(0)=1$$
$$y_2' = 3y_1 + 4y_2 \qquad y_2(0)=0$$

$$y' = Ay \qquad y(0)=\begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}y_1'\\y_2'\end{pmatrix}=\begin{pmatrix}2&1\\3&4\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}$$

Eulerova metoda (vlastních čísel)

$$u = h e^{\lambda x} \Rightarrow \underbrace{(A-\lambda E)}_{\text{singulární matice nám připouští } \exists h \neq 0} h = 0$$

vlastní čísla $\lambda_1, \lambda_2 \dots \lambda_n \dots \det(A-\lambda E)=0$

A – matice $n \times n$

pro $\lambda_1, \lambda_2 \dots \lambda_n \rightarrow u_1, \dots u_n$

$$y = c_1 u_1 + \dots + c_n u_n$$

• vlastní čísla:

$$\begin{vmatrix}2-\lambda & 1\\ 3 & 4-\lambda\end{vmatrix} = \lambda^2 - 6\lambda + 5 = 0$$

$\lambda_1 = +5$, $\lambda_2 = 1$ } jednoduché reálné kořeny

$\lambda_1 = 5$: $u_1 = h_1 \cdot e^{\lambda x} = h_1 \cdot e^{5x}$

h_1 – určíme jako řešení soustavy lin. rovnic:

$$(A - \lambda_1 E) h_1 = 0$$

$$\begin{pmatrix}2-5 & 1\\ 3 & 4-5\end{pmatrix}\begin{pmatrix}h_1\\h_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}-3 & 1\\ 3 & -1\end{pmatrix}\begin{pmatrix}h_1\\h_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

horní $\triangle$

$$\begin{pmatrix}-3 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}h_1\\h_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$-3h_1 + h_2 = 0$$
$$h_2 = 3h_1$$

volba h_1 nechť je $h_1 = 1 \Rightarrow h_2 = 3$

$$\Rightarrow h_1 = \begin{pmatrix}1\\3\end{pmatrix} \Rightarrow u_1 = \begin{pmatrix}1\\3\end{pmatrix} e^{5x}$$

$\lambda_2 = 1$: $u_2 = h_2 \cdot e^{1x}$

$$\begin{pmatrix}2-1 & 1\\ 3 & 4-1\end{pmatrix}\begin{pmatrix}h_1^*\\h_2^*\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}1 & 1\\ 3 & 3\end{pmatrix}\begin{pmatrix}h_1^*\\h_2^*\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

horní $\triangle$

$$\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}h_1^*\\h_2^*\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$h_1^* + h_2^* = 0$$

volba: $h_1 = 2 \Rightarrow h_2 = -2$

$$h_2 = \begin{pmatrix}2\\-2\end{pmatrix} \Rightarrow u_2 = \begin{pmatrix}2\\-2\end{pmatrix} e^x$$

$$y = c_1\begin{pmatrix}1\\3\end{pmatrix}e^{5x} + c_2\begin{pmatrix}2\\-2\end{pmatrix}e^x \qquad c_1, c_2 \in \mathbb{R}$$

$$y_1 = c_1 e^{5x} + 2c_2 e^x$$
$$y_2 = 3c_1 e^{5x} - 2c_2 e^x$$

$y_1(0) = 1$

$$1 = c_1 + 2c_2$$
$$0 = 3c_1 - 2c_2$$

$$1 = 4c_1 \Rightarrow c_1 = \frac{1}{4}$$

$$c_2 = \frac{1-\frac{1}{4}}{2} = \frac{3}{8}$$

$1 = 4c_1 - \dots$ $c_2 = \frac{3}{4}$

$$y = \begin{pmatrix}1/4\\3/4\end{pmatrix}e^{5x} + \begin{pmatrix}3/4\\-3/4\end{pmatrix}e^x$$

fázový portrét: