

$$(P.F.) \quad y' = \begin{pmatrix} 6 & -11 \\ 1 & -6 \end{pmatrix} y + \begin{pmatrix} 13 \cos x \\ -13 \sin x \end{pmatrix}$$

$$\begin{vmatrix} 6-\lambda & -11 \\ 1 & -6-\lambda \end{vmatrix} = \lambda^2 - 36 + 11 = \lambda^2 - 25 = 0$$

$$\lambda = \pm 5$$

$$\lambda_1 = 5$$

$$\begin{pmatrix} 1 & -11 \\ 1 & -11 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0 \rightarrow \begin{pmatrix} 1 & -11 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0$$

$$h_1 = 11h_2$$

$$v: h_2 = 1 \Rightarrow h_1 = 11 \quad h_1 = \begin{pmatrix} 11 \\ 1 \end{pmatrix}$$

$$u_1 = \begin{pmatrix} 11 \\ 1 \end{pmatrix} e^{5x}$$

$$\lambda_2 = -5$$

$$\begin{pmatrix} 11 & -11 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} h_1^* \\ h_2^* \end{pmatrix} = 0 \rightarrow \begin{pmatrix} 11 & -11 \\ 0 & 0 \end{pmatrix} h^* = 0$$

$$11h_1^* = 11h_2^*$$

$$v: h_1^* = 1 \Rightarrow h_2^* = 1 \quad h_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-5x}$$

$$y_H = c_1 \begin{pmatrix} 11 \\ 1 \end{pmatrix} e^{5x} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-5x}$$

$$f_1 = \begin{pmatrix} 13 \cos x \\ -13 \sin x \end{pmatrix} \quad b=1; a=0; \lambda^* = i \Rightarrow k=0, M=0$$

$$y_p = \begin{pmatrix} A \cos x + B \sin x \\ C \cos x + D \sin x \end{pmatrix}$$

$$y_p' = \begin{pmatrix} -A \sin x + B \cos x \\ -C \sin x + D \cos x \end{pmatrix}$$

$$f = e^{i0} [A \cos x + B \sin x]$$

$$f = e^{i0} \begin{pmatrix} 13 \\ 0 \end{pmatrix} \cos x + \begin{pmatrix} 0 \\ -13 \end{pmatrix} \sin x$$

$$P_0 \quad P_0^*$$

$$y' = Ay + f$$

$$(1) -A \sin x + B \cos x = 6A \cos x + 6B \sin x + 13 \cos x - 11A \sin x - 11B \sin x$$

$$(2) -C \sin x + D \cos x = C \cos x - 6D \sin x - 13 \sin x - 6C \cos x + D \sin x$$

$$\text{cos: } B = 6A + 13 - 11A$$

$$\text{cos: } D = C - 6C$$

$$D = -5C$$

$$\text{sin: } -A = 6B - 11B$$

$$\text{sin: } -C = -6D - 13 + D$$

$$13 = C - 5D$$

$$B + 5A = 13 \quad | \cdot 5$$

$$-5B + A = 65$$

$$26A = 65$$

$$A = -3 \quad C = 0$$

$$B = -5 \quad D = -3$$

$$y_p = \begin{pmatrix} -3 \cos x - 5 \sin x \\ -3 \sin x \end{pmatrix}$$

$$y = c_1 \begin{pmatrix} 11 \\ 1 \end{pmatrix} e^{5x} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-5x} + \begin{pmatrix} -3 \cos x - 5 \sin x \\ -3 \sin x \end{pmatrix}$$