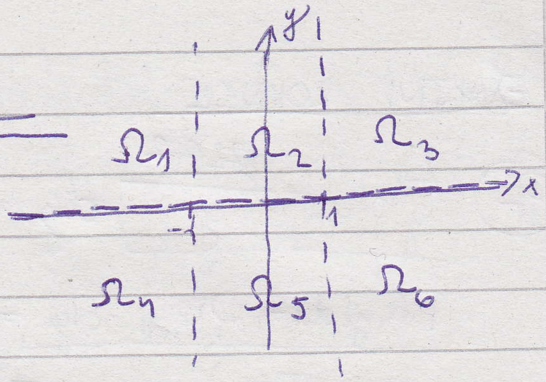
Př

$$y' - \frac{xy}{2(x^2-1)} = \frac{x}{2y}\ ; \quad y(2)=1$$



ex+jednozn.: $f(x,y) = \frac{xy}{2(x^2-1)} + \frac{x}{2y}$

$\frac{\partial f}{\partial y} = \frac{x}{2(x^2-1)} - \frac{x}{2y^2}$ $\quad y \neq 0$, $x \neq \pm 1$

$r=-1$; subst: $u = y^2$

$u' = 2y \cdot y'$

$y' - \frac{xy}{2(x^2-1)} = \frac{x}{2y}\ /\ 2y$ – přenásobit zadání

$2yy' - \frac{xy^2}{x^2-1} = x$

$u' - \frac{x}{x^2-1}u = x$ LODR1 nehomo.

① Přidružená homog.

$u_h' - \frac{x}{x^2-1}u_h = 0$

$\frac{du_h}{dx} = \frac{x}{x^2-1}u_h\ /\ \frac{dx}{du_h}$ $\quad u_h \neq 0$

$\int \frac{du_h}{u_h} = \frac{1}{2}\int \frac{2x\,dx}{x^2-1}$ $\quad \int \frac{f'}{f}dx = \ln|f|$

$\ln|u_h| = \frac{1}{2}\ln|x^2-1| + \ln C^*$; $C = \mathbb{R}$

$u_h = C \cdot \sqrt{|x^2-1|}$; $C \in \mathbb{R}$

I. $[x,y] \in \Omega_2 \cup \Omega_5$: $u_h = C \cdot \sqrt{1-x^2}$

II. $[x,y] \in \Omega_{1,3,4,6}$: $u_h = C \cdot \sqrt{x^2-1}$

$[x_0, y_0] = [2,1] \in \Omega_3$: $u_h = C\sqrt{x^2-1}$

② variace konst $[x_0,y_0] \in \Omega_3$:

$u = C(x) \cdot \sqrt{x^2-1}$

$u' = C'\sqrt{x^2-1} + C\frac{1}{2}\frac{2x}{\sqrt{x^2-1}}$ – dosazení:

$C'\sqrt{x^2-1} + \underbrace{C \cdot \frac{x}{\sqrt{x^2-1}} - \frac{x}{\sqrt{x^2-1}} C \cdot \sqrt{x^2-1}}_{=0} = x$

$\int dC = \int \frac{x\,dx}{\sqrt{x^2-1}}$

$C = \sqrt{x^2-1} + D$

$u = y^2 = (\sqrt{x^2-1} + D) \cdot \sqrt{x^2-1}$

$y = \pm\sqrt{D \cdot \sqrt{x^2-1} + x^2 - 1}$

(+) $\Omega_1 \cup \Omega_3$

(−) $\Omega_4 \cup \Omega_6$

$y(2) = 1$: $\sqrt{D\sqrt{3} + 3} = 1$

$D = \frac{-2}{\sqrt{3}}$

$y = \sqrt{-\frac{2}{3}\sqrt{x^2-1} + x^2 - 1}$